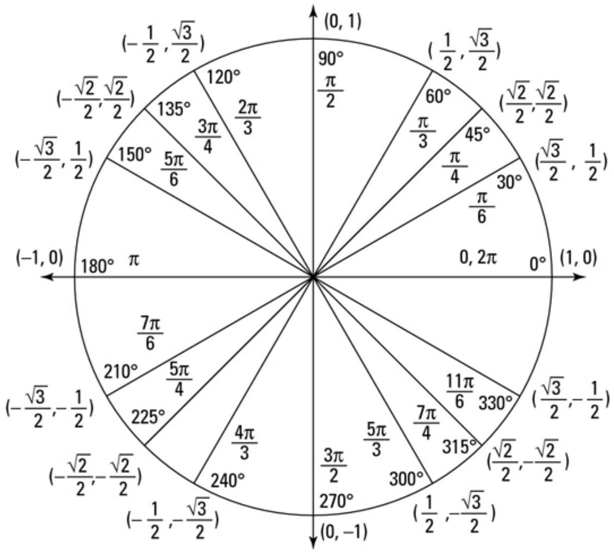
MATH 1060: Unit 1 Review Sheet

Quick Review



Things to Remember

- $\sqrt[a]{x^b} = x^{b/a}$
- SOHCAHTOA
- Conjugate of $\sqrt{a} \pm b$ is $\sqrt{a} \mp b$

1.3: Exponential Functions

$f(x) = b^x$; $b \neq 1$ and b is a positive number

domain: $(-\infty, \infty)$
range: $(0, \infty)$
natural exponential function: $f(x) = e^x$
Laws of Exponents:

- $a^{x+y} = a^x a^y$
- $a^{x-y} = \frac{a^x}{a^y}$
- $(a^x)^y = a^{xy}$
- $(ab)^x = a^x b^x$

1.3: Inverse Functions

inverse function: Given a function f, its inverse (if it exists) is a function f^{-1} such that whenever $y = f(x)$, then $f^{-1}(y) = x$
Properties of Inverse Functions:

- The domains and ranges of f and f^{-1} are switched.
- The graph of f^{-1} is the graph of f reflected about the line $y = x$.
- $f^{-1}(x)$ is the inverse of $f(x)$, but $(f(x))^{-1}$ is the reciprocal of $f(x)$.

Check for inverse: If the function passes the horizontal line test, the function is one-to-one and has an inverse
How to find the inverse:

1. Switch x and y.
2. Solve for y.
3. Substitute $f^{-1}(x)$ for y.

1.3: Logarithmic Functions

$$f(x) = \log_b(x)$$

Exp/Log: Exponential and logarithmic functions are inverses

- $b^{\log_b x} = x$ for $x > 0$
- $\log_b(b^x) = x$ for all x

Natural Logarithmic Function: $f(x) = \ln(x)$
Laws of Logarithms:

- $\log_b(xy) = \log_b(x) + \log_b(y)$
- $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
- $\log_b(x^r) = r\log_b(x)$

Change of Basis Rules:

- $b^x = e^{x\ln(b)}$
- $\log_b(x) = \frac{\ln(x)}{\ln(b)}$ for $x > 0$

1.4: Trig Functions and Inverses

Trigonometric Identities:

- $\csc(\theta) = \frac{1}{\sin(\theta)}$
- $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$
- $\sec(\theta) = \frac{1}{\cos(\theta)}$
- $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$
- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

Inverse trig functions take a *number* as an input and output an *angle*
Example: For $\cos^{-1}\left(\frac{1}{2}\right)$, Ask yourself, "What angle would give $cos(\theta) = \frac{1}{2}$?"
Important Note: Be careful that the angle you give is within the domain for the inverse trig function.

Function	Range
$y = \sin^{-1}(x)$	$y \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
$y = \cos^{-1}(x)$	$y \in [0, \pi]$
$y = \tan^{-1}(x)$	$y \in (-\frac{\pi}{2}, \frac{\pi}{2})$
$y = \csc^{-1}(x)$	$y \in (0, \frac{\pi}{2}] \cup (\pi, \frac{3\pi}{2}]$
$y = \sec^{-1}(x)$	$y \in [0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{3})$
$y = \cot^{-1}(x)$	$y \in (0, \pi)$

2.1: The Idea of Limits

Average Velocity$= v_{avg} = \frac{s(t_1)-s(t_0)}{t_1-t_0}$
Secant Line $= m_{sec} = \frac{s(t_1)-s(t_0)}{t_1-t_0}$

Note:

- average velocity $\rightarrow$ secant line
- instantaneous velocity $\rightarrow$ tangent line
- secant lines approach the tangent line
- slopes of secant lines approach the slops of the tangent line at the point $(a, s(a))$.

Inst Velocity $= v_{inst} = \lim_{t\to a} v_{avg} = \lim_{t\to a} \frac{s(t)-s(a)}{t-a}$
slope of tangent line$= m_{tan} = \lim_{t\to a} m_{sec}$
$= \lim_{t\to a} \frac{s(t)-s(a)}{t-a}$

2.2: Definitions of Limits

$$\lim_{x\to a} f(x) = L$$

If the limit exists, it depends on the value of f *near* a, *not* the value of $f(a)$.

Right-sided limit: $\lim_{x\to a^+} f(x) = L$

Left-sided limit: $\lim_{x\to a^-} f(x) = L$

Theorem 2.1: Assume f is defined for all x near a except possibly at a. Then

$$\lim_{x\to a} f(x) = L$$

if and only if

$$\lim_{x\to a^+} f(x) = L = \lim_{x\to a^-} f(x).$$

2.3: Computing Limits

Limit Laws: Assume $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist. The following properties hold, where c is a real number and $n > 0$ is an integer.

- $\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
- $\lim_{x\to a}(f(x)-g(x)) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
- $\lim_{x\to a}(cf(x)) = c\lim_{x\to a} f(x)$
- $\lim_{x\to a}(f(x)g(x)) = \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right)$
- $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, provided $\lim_{x\to a} g(x) \neq 0$
- $\lim_{x\to a}(f(x))^n = \left(\lim_{x\to a} f(x)\right)^n$
- $\lim_{x\to a}(f(x))^{1/n} = \left(\lim_{x\to a} f(x)\right)^{1/n}$, provided $f(x) > 0$, for x near a, if n is even.

Limits of Polynomial and Rational Functions: Assume p and q are polynomials and a is a constant.

- $\lim_{x\to a} p(x) = p(a)$
- $\lim_{x\to a} \frac{p(x)}{q(x)} = \frac{p(a)}{q(a)}$, provided $q(a) \neq 0$.

2.3: Computing Limits Continued

One Sided Limits: You can still use direct substituion!

Direct Sub Doesn't Work?
If you get $\frac{0}{0}$ I.F. by direct substitution, write $\frac{0}{0}$ I.F. then try the following:

- algebraically manipulate
- factor and cancel out terms
- multiply by the conjugate

The Squeeze Theorem: Assume the functions f, g, and h satisfy $f(x) \leq g(x) \leq h(x)$ for all values of x near a, except possibly at a. If $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$.

Important Inequalities:

- $-1 \leq \sin(\theta) \leq 1$
- $-1 \leq \cos(\theta) \leq 1$

How to Use Squeeze Theorem:

1. Use one of the two above inequalities
2. multiply/divide/subtract/add to all terms in the inequality to get the middle to look like what you want to take the limit of
3. Take the limit of the left hand side of the inequality
4. Take the limit of the right hand side of the inequality
5. If these limits match, then the limit of the middle is also the same

Useful Trig Limits:

- $\lim_{x\to 0} \sin(x) = 0$
- $\lim_{x\to 0} \cos(x) = 1$

2.4: Infinite Limits

$$\lim_{x\to a} f(x) = \infty \quad \text{OR} \quad \lim_{x\to a} f(x) = -\infty$$

Finding Infinite Limits:

1. Try direct substitution first.
2. if you get $\frac{0}{0}$, see section 2.3.
3. if you get $\frac{\text{nonzero number}}{0}$,
   - try plugging in numbers reeeeeeally close to the right and left of a.
   - if $\lim_{x\to a^-} f(x) = \infty = \lim_{x\to a^+} f(x)$, then $\lim_{x\to a} f(x) = \infty$. (Same for $-\infty$).

vertical asymptote: if $\lim_{x\to a} f(x) = \pm\infty$, $\lim_{x\to a^+} f(x) = \pm\infty$, or $\lim_{x\to a^-} f(x) = \pm\infty$, then the line $x = a$ is a vertical asymptote of f.

Finding Vertical Asymptotes:

1. Find the values where the denominator$= 0$ but the numerator$\neq 0$. You will usually have to factor.
2. Prove that you have a vertical asymptote using limits. Take the limit of the function as x approaches each value from the left and right. At least one limit should be infinite.

2.5 Limits at Infinity

$$\lim_{x\to\infty} f(x) = L$$

horizontal asymptote: The line $y = b$ is a *horizontal asymptote* of the curve $y = f(x)$ if either

$$\lim_{x\to\infty} f(x) = b \qquad \text{OR} \qquad \lim_{x\to-\infty} f(x) = b$$

I.F.: $\frac{0}{0}, \frac{\infty}{\infty}, 0\cdot\infty, \infty-\infty, 0^0, 1^\infty, \infty^0$

Limits at Infinity of Powers and Polynomials:

- $\lim_{x\to\infty} x^n = \infty$ and $\lim_{x\to-\infty} x^n = \infty$; n is *even*.
- $\lim_{x\to\infty} x^n = \infty$ and $\lim_{x\to-\infty} x^n = -\infty$; n is *odd*.
- $\lim_{x\to\pm\infty} \frac{1}{x^n} = \lim_{x\to\pm\infty} x^{-n} = 0$
- $\lim_{x\to\pm\infty} p(x) = \infty$ or $-\infty$ depends on the degree of the polynomial and sign of leading coefficient.

2.5: Limits at Infinity (Continued)

Technique for Rational Functions:

1. Choose the highest power of x in the denominator.
2. Divide every term in the numerator and denominator by the highest power of x in the denominator.
3. Take the limit of each term. Recall that the limit as $x \to \pm\infty x^{-n} = 0$

VERY IMPORTANT NOTE:

- $\sqrt{x^2} = |x| = x$ if $x > 0$ (when $x \to \infty$)
- $\sqrt{x^2} = |x| = -x$ if $x < 0$ (when $x \to -\infty$)

slant asymptote: When the degree of numerator is *ONE MORE* than degree of denominator,

1. Use long division to divide the numerator by the denominator.
2. The equation of the line that is the slant asymptote is the quotient from your long division.

2.6: Continuity

continuous at a point: A function f is continuous at a number a if $\lim_{x\to a} f(x) = f(a)$.

Conditions for Continuity of f at a:

- $f(a)$ is defined. (a is in the domain of f)
- $\lim_{x\to a} f(x)$ exits
- $\lim_{x\to a} f(x) = f(a)$

Types of Discontinuities:

- removable
- jump
- infinite
- oscillating

continuous from the right at a number a: A function f is continuous from the right at a number a if

$$\lim_{x\to a^+} f(x) = f(a)$$

continuous from the left at a number b: A function f is continuous from the left at a number b if

$$\lim_{x\to b^-} f(x) = f(b)$$

continuous on an interval: A function is continuous on an interval if it is continuous at every number in the interval

continuous function: A continuous function is continuous at every point of its domain.

Continuity Theorems:

- The inverse of a continuous function is continuous.
- The composition of continuous functions is continuous.

2.6: Continuity (Continued)

Functions that are Continuous of their Domains

- polynomials $(-\infty, \infty)$
- rational functions (everywhere except denominator=0)
- root functions (inside of root ≥ 0)
- trig functions
- inverse trig functions
- exponential functions $(-\infty, \infty)$

The Intermediate Value Theorem (IVT)

If

- f continuous on $[a, b]$
- $f(a) < L < f(b)$

Then,

- $a < c < b$
- $f(c) = L$

2.7: $\delta - \epsilon$ Proof

limit: If for every number $\epsilon > 0$, there is a number $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \epsilon$.

$$\lim_{x\to a} f(x) = L$$

Steps for Proving $\lim_{x\to a} f(x) = L$

1. Write down what $f(x), L$, and a are.
2. Find δ in your scratch work using $|f(x) - L| < \epsilon$. This is not part of your proof and δ should be in terms of ϵ. We try to algebraically get $|f(x) - L|$ to look like a multiple of $|x - a|$.
3. Write your proof using the following sentences with your values for δ, a, and L plugged in:
   - Given $\epsilon > 0$, let $\delta =$
   - If $0 < |x - a| < \delta =$, then *SCRATCH WORK to show $|f(x) = L| < \epsilon$*.
   - By the definition of a limit, $\lim_{x\to a} f(x) = L$.

3.1: Introducing the Derivative

derivative: the slope of the tangent line

How to find equation of tangent line:

1. Find slope of the tangent line using:

$$m_{tan} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$
OR
$$m_{tan} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

2. Use point-slope formula to find equation

$$y - y_1 = m_{tan}(x - x_1)$$

Note that $y_1 = f(x_1)$ if not given

How to find equation of normal line:

1. Find slope of the tangent line using:

$$m_{tan} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$
OR
$$m_{tan} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

2. Find the slope of the normal line using:

$$m_{norm} = -\frac{1}{m_{tan}}$$

3. Use point-slope formula to find equation

$$y - y_1 = m_{norm}(x - x_1)$$

Note that $y_1 = f(x_1)$ if not given

position: $s(t)$

velocity: derivative of position

$$v(a) = s'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

3.2: The Derivative as a Function

derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

How to find equation of tangent line:

1. Find derivative of function
2. Plug in given x_1 value to derivative to get slope of tangent line

$$m_{tan} = f'(x_1)$$

3. Use point-slope formula to find equation

$$y - y_1 = m_{tan}(x - x_1)$$

Note: $y_1 = f(x_1)$ if not given

differentiable at a If $f'(a)$ exists

differentiable on an open interval if f is differentiable at every number in the interval.

Theorem:

- Differentiability $\implies$ Continuity
- Continuity $\not\Longrightarrow$ Differentiability

Differentiability Fails:

- discontinuity
- corner
- vertical tangent, cusps

$f(x)$	$f'(x)$
increasing	positive
decreasing	negative
horizontal tangent	zero (root)
not diff at a	$f'(a)$ is undefined

Graph Note: The derivative of a graph of degree n is a graph of degree $n-1$. So, the derivative is one degree lower than the original function.

3.2: The Derivative as a Function (Continued)

Functions and their Derivative Graphs

- Look for *horizontal tangent lines* first and match these x-coordinates to *zeros* on the derivative graph.
- Look for *points of discontinuity* and match these to *holes or gaps* in the derivative graph.
- Look for other values of x where the function is *not differentiable*. The derivative graph will *not be defined* there.
- Look for the intervals of *increase* on the original graph. This tells you when the derivative graph is *above the x-axis*.
- Look for the intervals of *decrease* on the original graph. This tells you when the derivative graph is *below the x-axis*.

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3.3: Rules of Differentiation

- $\frac{d}{dx}(c) = 0$
- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}(cf(x)) = cf'(x)$
- $\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$

3.3: Exponential Differentiation

- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(e^{g(x)}) = e^{g(x)}g'(x)$

3.4: The Product Rule

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = \left[\frac{d}{dx}f(x)\right]g(x) + f(x)\left[\frac{d}{dx}g(x)\right]$$

or

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

3.4: The Quotient Rule

If f and g are both differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\left[\frac{d}{dx}f(x)\right]g(x) - f(x)\left[\frac{d}{dx}g(x)\right]}{[g(x)]^2}$$

or

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

3.7: The Chain Rule

- $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$
  1. Take the derivative of the outside function and leave the inside unchanged
  2. Multiply by the derivative of the inside function

3.6: Marginal Cost

- $C(x)$ is the total cost to produce x units
- **Average cost:** $\frac{C(x)}{x}$
- **additional cost:** $\Delta C = C(x_2) - C(x_1)$
- **marginal cost:** $C'(n) \approx C(n+1) - C(n)$

3.6: Rates of Change

instantaneous rate of change: the derivative; also the limit of the average rates of change
elapsed time: Δt
displacement: $\Delta s = f(a + \Delta t) - f(a)$
average velocity: $\frac{\Delta s}{\Delta t}$
velocity: the derivative of position with respect to time

$$v(t) = s'(t)$$

Note: The sign of velocity indicates direction
speed: the magnitude of the velocity

$$\text{speed} = |v(t)|$$

acceleration: the derivative of velocity with respect to time

$$a(t) = v'(t) = s''(t)$$

Note:

- Speeding up: velocity and acceleration have the same sign
- Slowing down: velocity and acceleration have opposite signs

Note: To find when an object is at rest, set velocity equal to 0 and solve for t.
Free Fall:

- The object reaches its maximum height when velocity is 0.
  1. set $v(t) = 0$
  2. solve for t. This is the time the object reaches its maximum height
  3. plug in the time you found into $s(t)$ to get the maximum height
- The object hits the ground when the height=0.
  1. Find the time the object hits the ground by solving $s(t) = 0$
  2. Find the velocity with which the object hits the ground by plugging in time to velocity function.

3.5: Special Trig Limits

- $\lim_{x \to 0} \frac{\sin x}{x} = 1$
- $\lim_{x \to 0} \frac{\cos x - 1}{x} = 0$

Note: You need to make sure the correct multipliers in front of x are in place to use these special limits. You CANNOT use L'Hopital's Rule yet!

3.5: Trig Derivatives

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\cot x) = -\csc^2 x$
- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\csc x) = -\csc x \cot x$

3.10: Inverse Trig Derivatives

- $\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\left[\cos^{-1} x\right] = \frac{-1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2}$
- $\frac{d}{dx}\left[\csc^{-1} x\right] = \frac{-1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}\left[\sec^{-1} x\right] = \frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}\left[\cot^{-1} x\right] = \frac{-1}{1+x^2}$

Note: Do not forget CHAIN RULE!! You must multiply by the derivative of the inside!
i.e.

$$\frac{d}{dx}\left[\sin^{-1}(g(x))\right] = \frac{1}{\sqrt{1-g(x)^2}} \cdot g'(x)$$

3.8: Implicit Differentiation

implicit form: an equation that is not solved for one variable

Implicit Differentiation Method:

1. Differentiate both sides with respect to x
2. Solve for $\frac{dy}{dx}$ (or y')

Note: You need to apply the chain rule for terms involving y!

Examples:

- $\frac{d}{dx}[y] = \frac{dy}{dx} = y'$
- $\frac{d}{dx}[y^2] = 2y\frac{dy}{dx} = 2yy'$

Implicit Differentiation for Second Derivatives

1. First find $\frac{dy}{dx}$
2. Differentiate $\frac{dy}{dx}$
3. Solve for $\frac{d^2y}{dx^2}$
4. Substitute $\frac{dy}{dx}$ into $\frac{d^2y}{dx^2}$

3.9: Exponential Function with Base a

$$\frac{d}{dx}(a^x) = a^x \ln a$$
$$\frac{d}{dx}(a^{g(x)}) = a^{g(x)} g'(x) \ln a$$

Note: Remember you can always use logarithmic differentiation for these if you can't remember this rule.

3.9: Derivatives of Logarithmic Functions

- $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- $\frac{d}{dx}(\ln |x|) = \frac{1}{x}$
- $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$ OR $\frac{d}{dx}[\ln(g(x))] = \frac{g'(x)}{g(x)}$

3.9: Logarithmic Differentiation

Logarithmic Differentiation:
We use Log Diff when we have a variable in our base and a variable in our exponent

1. Take ln of both sides
2. Use the properties of $\ln(x)$ to simplify
3. Use implicit differentiation to take derivative of both sides
4. Isolate y' (or $\frac{dy}{dx}$)
5. Substitute y into result.

Recall Laws of Logarithms:

- $\ln(xy) = \ln(x) + \ln(y)$ for $x > 0, y > 0$
- $\ln(\frac{x}{y}) = \ln(x) - \ln(y)$ for $x > 0, y > 0$
- $\ln(x^p) = p\ln(x)$ for $x > 0$ and $p \in \mathbb{R}$

3.11: Related Rates

Method:

1. Draw a diagram
2. introduce notation and include units
3. express the given information and the required rate in terms of derivatives
4. write an equation that relates the various quantities
5. use implicit differentiation and the chain rule to differentiate both sides of the equation with respect to time
6. substitute the given info into the result
7. solve for the unknown rate
8. write a summary sentence

4.1: Maxima and Minima

abs max: at c if $f(c) \geq f(x)$ for all x in domain
abs min: at c if $f(x) \leq f(x)$ for all x in domain
local max: at c if $f(c) \geq f(x)$ when x is near c
local min: at c if $f(x) \leq f(x)$ when x is near c
Extreme Value Theorem: A continuous function on $[a, b]$ has an absolute max and an absolute min on $[a, b]$.
Closed Interval Method: (finding absolute extrema)

1. Find the critical points of f
2. Evaluate f at the critical points **AND** the endpoints
3. largest function value = abs max
   smallest function value = abs min

4.2: Mean Value Theorem

Rolle's Theorem: If $y = f(x)$ is

i) continuous on $[a, b]$
ii) differentiable on (a, b)
iii) $f(a) = f(b)$

then there is a number c in (a, b) such that $f'(c) = 0$.

Mean Value Theorem: If $y = f(x)$ is:

i) continuous on $[a, b]$
ii) differentiable on (a, b)

then there is a number c in (a, b) such that
$f'(c) = \frac{f(b)-f(a)}{b-a}$

Things to Remember

Find where the tangent line is horizontal:

1. Find derivative
2. Set derivative equal to 0
3. Solve for x

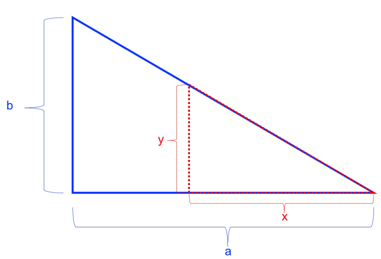
Find the equation of tangent line:

1. Find derivative.
2. If you are not given both x_1 and y_1 of the point of tangency, calculate y_1 by evaluating the ORIGINAL function at x_1.
3. Evaluate derivative at the point of tangency, (x_1, y_1). This is your slope of the tangent line, m_{tan}
4. Plug x_1, y_1, m_{tan} into the point slope formula $y - y_1 = m_{\text{tan}}(x - x_1)$ and solve for y.

Find the equation of normal line:

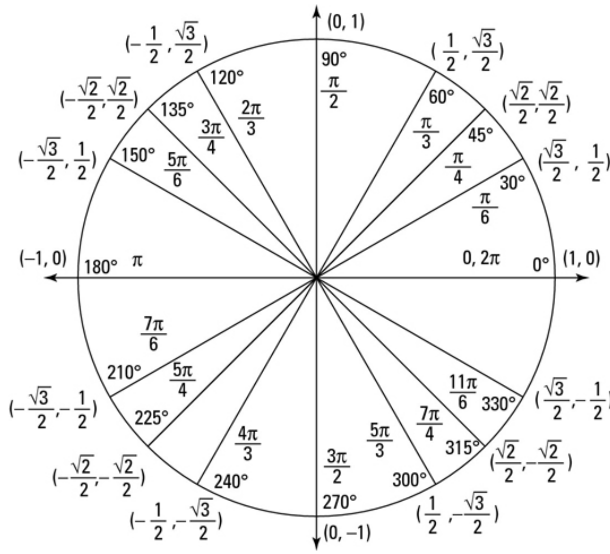
1. Find derivative.
2. If you are not given both x_1 and y_1 of the point of tangency, calculate y_1 by evaluating the ORIGINAL function at x_1.
3. Evaluate derivative at the point of tangency, (x_1, y_1). This is your slope of the tangent line, m_{tan}. To get the slope of the normal line, take the negative reciprocal of m_{tan}. $m_{\text{norm}} = \frac{-1}{m_{\text{tan}}}$
4. Plug $x_1, y_1, m_{\text{norm}}$ into the point slope formula $y - y_1 = m_{\text{norm}}(x - x_1)$ and solve for y.

Similar Triangles



$$\frac{a}{b} = \frac{x}{y}$$

Useful PreCalculus



Things to Remember

- $\sqrt[a]{x^b} = x^{b/a}$
- SOHCAHTOA

Common Trig Identities

- $\csc(x) = \frac{1}{\sin(x)}$
- $\sec(x) = \frac{1}{\cos(x)}$
- $\cot(x) = \frac{1}{\tan(x)}$
- $\sin^2(x) + \cos^2(x) = 1$
- $\tan(x) = \frac{\sin(x)}{\cos(x)}$

MATH 1060: Exam 3 Review Sheet

4.3: 1st Derivatives and Shapes of a Graphs

Remember:

- If $f'(x) > 0$, then $f(x)$ is increasing.
- If $f'(x) < 0$, then $f(x)$ is decreasing

Increasing and Decreasing Test:

1. Find x values where either
   - $f'(x) = 0$
   - $f'(x)$ DNE
2. Make a sign chart!
3. Determine:
   - $f'(x) > 0 \implies f$ increasing
   - $f'(x) < 0 \implies f$ decreasing

local minimum: If $f'(x)$ changes from $-$ to $+$ at a critical point c, then $f(x)$ has a local minimum at $x = c$.
local maximum: If $f'(x)$ changes from $+$ to $-$ at a critical point c, then $f(x)$ has a local maximum at $x = c$
One Local $\implies$ Absolute: Suppose f is continuous one an interval that contains exactly one local extremum at c.

- If a local min occurs at c, then $f(c)$ is the absolute min of f on the interval.
- If a local max occurs at c, then $f(c)$ is the absolute max of f on the interval.

4.3: 2nd Derivatives and Shapes of a Graphs

Remember:

- If $f''(x) > 0$, then $f(x)$ is concave UP.
- If $f''(x) < 0$, then $f(x)$ is concave DOWN.

Concavity Test:

1. Find x values where
   - $f''(x) = 0$
   - $f''(x)$ DNE
2. Make a sign chart!
3. Determine:
   - $f''(x) > 0 \implies f(x)$ concave UP
   - $f''(x) < 0 \implies f(x)$ concave DOWN

Inflection Point: a point on the graph where the concavity CHANGES
Second Derivative Test: Suppose that $f''(x)$ is continuous near $x = c$ with $f'(c) = 0$. Then,

- $f''(c) > 0 \implies f(x)$ has a local min at $x = c$.
- $f''(c) < 0 \implies f(x)$ has a local max at $x = c$
- $f''(c) = 0 \implies$ inconclusive

4.5: Optimization

1. Define all variables.
2. Draw a picture!
3. State function to be optimized.
4. State constraints.
5. Write function as one variable (use constraints).
6. Find domain of function.
7. Find absolute extrema.
8. Verify absolute extrema.
9. Write a sentence!

4.4: Graphing Functions

1. Identify domain
2. Find intercepts
   - y-intercept: set $x = 0$ and solve for y
   - x-intercept: set $y = 0$ and solve for x
3. Check for Symmetry
   - y-axis: $f(-x) = f(x)$
   - origin: $f(-x) = -f(x)$
   - periodic: $\sin(x), \cos(x)$, etc.
4. Asymptotes
   - horizontal: if $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$
   - vertical: points where denominator $= 0$.
   - slant: If degree of numerator is one more than degree of denominator, use long division to find quotient.
5. Increasing/Decreasing:
   - Find $f'(x)$ and make sign chart
6. Find Local Extrema
7. Find Concavity and Inflection Points
8. Sketch Graph!

4.6: Linearization and Differentials

Linearization: $L(x) = f(a) + f'(a)(x - a)$

1. Choose a value of a to produce a small error
2. Find $f(a)$, $f'(x)$, and $f'(a)$
3. Create $L(x)$ using formula
4. Plug in the quantity you want to calculate into $L(x)$

Differential: $dy = f'(a)dx$

1. Take derivative of given function
2. Plug in known values

4.7: L'Hopital's Rule

Indeterminate Forms for L'Hopital's: $\frac{0}{0}$, $\frac{\infty}{\infty}$
L'Hopital's Rule: $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \lim\limits_{x\to a} \frac{f'(x)}{g'(x)}$

- Apply when the limit gives you $\frac{0}{0}$ or $\frac{\infty}{\infty}$
- Take derivative of numerator and denominator SEPARATELY (no quotient rule!)

Related Indeterminate Forms: $\infty \cdot 0$, $\infty - \infty$

- For these indeterminate forms, try to use algebra to evaluate the limits.

Indeterminate Powers: 1^∞, 0^0, ∞^0

- Use e and $\ln(x)$ to change the form of the function:
$$\lim_{x\to a} f(x) = \lim_{x\to a} e^{\ln(f(x))}$$

1. Evaluate $\lim\limits_{x\to a} \ln(f(x)) = L$
2. Exponentiate Step 1: $\lim\limits_{x\to a} f(x) = e^L$

4.9: Antiderivatives

Antiderivative: F is the antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.
Theorem: If F is an antiderivative of f, then the most general antiderivative of f is $F(x) + C$
Indefinite Integral:

$$\int f(x)dx = F(x) + C$$

Power Rule: If exponent $\neq 1$:

1. Add 1 to exponent
2. Divide by new exponent

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Constant Multiple Rule:

$$\int af(x)dx = a\int f(x)dx$$

Sum Rule:

$$\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$$

Integral of $\frac{1}{x}$:

$$\int \frac{1}{x}dx = \ln|x| + C; x \neq 0$$

Initial Value Problem:

1. Find antiderivative (don't forget $+C$!)
2. Plug in given x value in antiderivative and set equal to given function value
3. Solve for C.
4. Write final solution with the value found for C.

Rectilinear Motion:

- $v(t) = \int a(t)dt$
- $s(t) = \int v(t)dt$
- Initial Conditions: $s(0)$ and $v(0)$
- $g = 9.8m/s^2$ or $g = 32ft/s^2$

5.1: Areas, Distances, Riemann Sum

Displacement: displacement= velocity $\times$ time

- If velocity is alway positive, displacement is the distance traveled.
- Find displacement by finding the area under the curve of velocity function.

Approximating Areas by Riemann Sums

1. Divide interval $[a, b]$ into n subintervals of equal length.
   - $x_0 = a$ and $x_n = b$
   - Length of each subinterval: $\Delta x = \frac{b-a}{n}$
2. Choose a point in each subinterval, x_k^*, and make a rectangle whose height is the function evaluated at that point, $f(x_k^*)$.
   - We usually choose x_k^* as left endpoint, right endpoint, or midpoint.
     - Left: $x_k^* = a + (k-1)\Delta x$
     - Right: $x_k^* = a + k\Delta x$
     - Midpoint: $\overline{x_k} = a + (k - \frac{1}{2})\Delta x$
   - area of k^{th} rectangle: $f(x_k^*) \cdot \Delta x$
3. Add together all the areas of the n rectangles.
   - $\sum\limits_{k=1}^{n} f(x_k^*)\Delta x =$
     $f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_n^*)\Delta x$

Common Sum Rules:

- $\sum\limits_{k=1}^{n} ca_k = c\sum\limits_{k=1}^{n} a_k$
- $\sum\limits_{k=1}^{n} (a_k + b_k) = \sum\limits_{k=1}^{n} a_k + \sum\limits_{k=1}^{n} b_k$
- $\sum\limits_{k=1}^{n} c = n \cdot c$

5.2: The Definite Integral

Net Area: the net area of the region bounded by a continuous function f and the x-axis between $x = a$ and $x = b$ is:

Area Above x-axis - Area Below x-axis
$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

Total Area: the total area of the region bounded by a continuous function f and the x-axis between $x = a$ and $x = b$ is:

Area Above x-axis + Area Below x-axis
$$\int_a^b |f(x)|dx$$

Properties of Definite Integrals:

- $\int_a^a f(x)dx = 0$
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$
- $\int_a^b kf(x)dx = k\int_a^b f(x)dx$ for a constant k
- $\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$
- $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b$
- $\int_a^b f(x)dx = \int_a^c f(x)dx - \int_b^c f(x)dx$

5.2: Definite Integrals with Riemann Sums

Calculate Definite Integral Using Riemann Sums:

1. Find $\Delta x = \frac{b-a}{n}$
2. Find an expression for the right endpoint /left endpoint/midpoint of the k^{th} subinterval
   - Left: $x_k = a + (k-1)\Delta x$
   - Right: $x_k = a + k\Delta x$
   - Midpoint: $x_k = a + (k - \frac{1}{2})\Delta x$
3. Find $f(x_k)$ by plugging in what you found for x_k everywhere there is an x in the function.
4. State the right/left/midpoint Riemann Sum, $\sum_{k=1}^{n} f(x_k)\Delta x$. This sum should be in terms of k and n.
5. Simplify the Riemann Sum using sum formulas given. The final answer should only be in terms of n.
6. Find the exact value of the definite integral by taking the limit of the simplified Riemann Sum

Sum Formulas: (will be given)

- $\sum_{k=1}^{n} c = cn$
- $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
- $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

5.3: The Fundamental Theorem of Calculus

FTOC Part I: If f is continuous on $[a, b]$, then the area function

$$A(x) = \int_a^x f(t)dt$$

for $a \le x \le b$ is continuous on $[a, b]$ and differentiable on (a, b). The area function satisfies $A'(x) = f(x)$:

$$A'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

Derivative of Integrals: If the lower limit is a constant and the upper limit is a function of x, use chain rule along with FTOC Part I:

$$\frac{d}{dx}\left(\int_a^{g(x)} f(t)dt\right) = f(g(x))g'(x)$$

FTOC Part II: If f is continuous on $[a, b]$ and F is any antiderivative of f, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Total Area Revisited:

1. Find x values where $f(x) = 0$ [x-intercepts].
2. Divide into subintervals using the x-intercepts.
3. Integrate f over each subinterval and add the absolute values.

MATH 1060: Unit 4 Review

Useful Trig Derivatives

- $\frac{d}{dx}(\sin(x)) = \cos(x)$
- $\frac{d}{dx}(\cos(x)) = -\sin(x)$
- $\frac{d}{dx}(\tan(x)) = \sec^2(x)$
- $\frac{d}{dx}(\cot(x)) = -\csc^2(x)$
- $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$
- $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$

Useful Inverse Trig Derivatives

- $\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\left[\cos^{-1} x\right] = \frac{-1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2}$
- $\frac{d}{dx}\left[\csc^{-1} x\right] = \frac{-1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}\left[\sec^{-1} x\right] = \frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}\left[\cot^{-1} x\right] = \frac{-1}{1+x^2}$

Note: Do not forget CHAIN RULE!! You must multiply by the derivative of the inside!
i.e.

$$\frac{d}{dx}\left[\sin^{-1}(g(x))\right] = \frac{1}{\sqrt{1-g(x)^2}} \cdot g'(x)$$

5.4: Working with Integrals

Theorem: Let a be a positive real number and let f be an integrable function on $[-a, a]$.

- If f is even, $\displaystyle\int_{-a}^{a} f(x)dx = 2\int_0^2 f(x)dx$
- If f is odd, $\displaystyle\int_{-a}^{a} f(x)dx = 0$

even function: symmetric about y-axis; $f(-x) = f(x)$
odd function: symmetric about origin; $f(-x) = -f(x)$

Average Value: $\bar{f} = \frac{1}{b-a}\displaystyle\int_a^b f(x)dx$

MVT for integrals: Let f be continous on $[a, b]$. There exists a point c in (a, b) such that

$$f(c) = \bar{f} = \frac{1}{b-a}\int_a^b f(x)dx$$

5.5: U-Substitution

Indefinite Integrals:

1. Identify u such that a constant multiple of du (derivative of u) appears in the integrand.
2. Substitute u and $du = u'dx$ into the integral.
3. Evaluate the new indefinite integral with respect to u. Don't forget your $+C$.
4. Replace u with the function of x, so your final answer is in terms of x.

Definite Integrals:

1. Identify u such that a constant multiple of du appears in the integrand.
2. Change your bounds of integration by plugging in your original a and b into your function of u.
3. Substitute u and $du = u'dx$ and the new bounds, $u(a)$ and $u(b)$, into the integral.

3\. Evaluate the new definite integral like normal. You do NOT have to make any substitutions to get in terms of x in the definite integral case since you have changed your bounds.